

FORM FACTORS IN AN ALGEBRAIC MODEL OF THE NUCLEON

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Abstract

We study the electromagnetic form factors of the nucleon in a collective model of baryons. In an algebraic approach to hadron structure, we derive closed expressions for both elastic and transition form factors, and consequently for the helicity amplitudes that can be measured in electro- and photoproduction.

1 Introduction

Currently there is a lot of interest in the structure of the nucleon and its resonances [1]. The (re)measurement of the electromagnetic form factors of baryon resonances forms an important part of the experimental program at various facilities, *e.g.* MAMI and CEBAF.

Extensive calculations of these observables were carried out in the relativized quark potential model [2], in which a relatively small number of low-lying configurations in the confining potential contribute significantly to the baryon wave functions. On the other hand, flux-tube models, soliton models, as well as some regularities in the observed spectrum (linear Regge trajectories and parity doubling) hint that an alternative, more collective type of dynamics may play a role in the structure of baryons.

In this contribution we discuss the electromagnetic couplings in a collective model using an algebraic approach for baryons.

2 Algebraic model of the nucleon

In [3] we introduced an algebraic model, in which the nucleon has the string configuration of Figure 1. Its three constituent parts are characterized by the internal degrees of freedom of spin, flavor and color and by the two relative Jacobi coordinates, $\vec{\rho}$ and $\vec{\lambda}$, and their conjugate momenta. For these six spatial degrees of freedom we suggested to use a $U(7)$ spectrum generating algebra whose building blocks are six dipole bosons, $b_{\rho,i}^\dagger$ and $b_{\lambda,i}^\dagger$ ($i = x, y, z$), and an auxiliary scalar boson, s^\dagger . For a system of interacting bosons the model space is spanned by the symmetric irreps $[N]$ of $U(7)$, which contains the oscillator shells with $n = 0, 1, 2, \dots, N$. Here N is the conserved total number of bosons. The full algebraic structure is obtained by combining the geometric part, $U(7)$, with the internal spin-flavor-color part, $SU_{sf}(6) \otimes SU_c(3)$.

For the nucleon (isospin $I = 1/2$) and delta ($I = 3/2$) families of resonances the three strings of Figure 1 have equal length and equal relative angles. Hence this configuration is an oblate top and has D_{3h} point group symmetry. The classification under D_{3h} is equivalent to the classification under permutations and parity. States are characterized by $(v_1, v_2); K, L_t^P$, where (v_1, v_2) denote the vibrations (stretching, bending); K denotes the projection of the rotational angular momentum L on the body-fixed symmetry axis, P the parity and t the symmetry type of the state under D_3 (a subgroup of D_{3h} isomorphic to the S_3 permutation group). The symmetry type of the geometric part must be the same as that of the spin-flavor part (the color part is antisymmetric). Therefore, one can use the representations of either D_3 or $SU_{sf}(6)$ to label the states: $A_1 \leftrightarrow 56$, $A_2 \leftrightarrow 20$, $E \leftrightarrow 70$.

In [3] we used a D_3 -invariant mass operator consisting of spatial and spin-flavor contributions to obtain a description of the mass spectrum of nonstrange baryons with a r.m.s. deviation of 39 MeV. In this collective model of the nucleon baryon resonances are interpreted as vibrations and rotations of an oblate symmetric top. The corresponding wave functions, when expressed in a harmonic oscillator basis, are spread over many shells and hence are truly collective.

3 Electromagnetic form factors

Helicity amplitudes and form factors can be measured in photo- and electroproduction of baryon resonances. In the nonrelativistic limit the transverse coupling of the photon field to the three constituent parts is given by a magnetic and an electric contribution

$$\mathcal{H} = 6\sqrt{\pi/k_0} \mu_3 e_3 \left[k s_{3,+} \hat{U} - \hat{T}_+ / g_3 \right]. \quad (1)$$

Here $\vec{k} = k\hat{z}$ is the photon momentum, k_0 the photon energy and s_3 , e_3 , g_3 , m_3 , $\mu_3 = eg_3/2m_3$ denote the spin, charge, g -factor, mass and the scale magnetic

moment of the third constituent, respectively. The operators \hat{U} and \hat{T}_+ only act on the spatial part of the baryon wave function. In an algebraic treatment they are given by

$$\begin{aligned}\hat{U} &= e^{-ik\beta\hat{D}_{\lambda,z}/X_D}, \\ \hat{T}_+ &= \frac{im_3k_0\beta}{2X_D} \left(\hat{D}_{\lambda,+} \hat{U} + \hat{U} \hat{D}_{\lambda,+} \right),\end{aligned}\quad (2)$$

where $\hat{D}_{\lambda,m} = (b_\lambda^\dagger \times s - s^\dagger \times \tilde{b}_\lambda)_m^{(1)}$ is the dipole operator in $U(7)$ which has the same transformation properties as the Jacobi coordinate λ_m . The coefficient X_D is a normalization factor and β represents the scale of the coordinate.

Since \hat{D}_λ is a generator of the algebra of $U(7)$, the matrix elements of \hat{U} are representation matrix elements of $U(7)$, *i.e.* generalized Wigner \mathcal{D} -matrices. By making an appropriate basis transformation they can be obtained exactly. However, in the limit of $N \rightarrow \infty$ (infinitely large model space) they can also be derived in closed form. This derivation consists of several steps. The rotational states $|K, L, M\rangle$ of the ground state band of the oblate symmetric top can be obtained by projection from an intrinsic (or coherent) state

$$\begin{aligned}|K, L, M\rangle &= \sqrt{\frac{2L+1}{8\pi^2}} \int d\Omega \mathcal{D}_{MK}^{(L)*}(\Omega) \mathcal{R}(\Omega) |N, R\rangle, \\ |N, R\rangle &\propto \left(s^\dagger + R(b_{\lambda,x}^\dagger + b_{\rho,y}^\dagger)/\sqrt{2} \right)^N |0\rangle.\end{aligned}\quad (3)$$

The coefficient R appears in the mass operator and is associated with the size of the string. Next we construct states with good D_3 symmetry by taking the linear combinations

$$\begin{aligned}|\psi_1\rangle &= \frac{1}{\sqrt{2(1+\delta_{K0})}} \left[(-)^L |K, L, M\rangle + |-K, L, M\rangle \right], \\ |\psi_2\rangle &= \frac{i}{\sqrt{2(1+\delta_{K0})}} \left[|K, L, M\rangle - (-)^L |-K, L, M\rangle \right].\end{aligned}\quad (4)$$

For $K(\bmod 3) = 0$ the wave function $|\psi_1\rangle$ is symmetric ($A_1 \leftrightarrow 56$) and $|\psi_2\rangle$ antisymmetric ($A_2 \leftrightarrow 20$), whereas for $K(\bmod 3) \neq 0$ the wave functions $|\psi_1\rangle$ and $|\psi_2\rangle$ are the two components of the mixed-symmetry doublet ($E \leftrightarrow 70$). Eq. (4) is consistent with the choice of geometry in $|N, R\rangle$. Finally, the matrix elements of \hat{U} and \hat{T}_+ can be derived in closed form in the $|K, L, M\rangle$ basis. For example, for \hat{U} we find

$$\begin{aligned}\langle K, L, M | \hat{U} | K', L', M' \rangle &= i^{K-K'} \sqrt{(2L'+1)/(2L+1)} \\ &\sum_\lambda \frac{1}{2} \left[1 + (-1)^{\lambda+K-K'} \right] (2\lambda+1) j_\lambda(k\beta) \langle L', M', \lambda, 0 | L, M \rangle \\ &\langle L', K', \lambda, K-K' | L, K \rangle \frac{\sqrt{(\lambda+K-K')!(\lambda-K+K')!}}{(\lambda+K-K')!(\lambda-K+K')!},\end{aligned}\quad (5)$$

where we have used that in the large N limit the intrinsic matrix element becomes diagonal in the orientation Ω of the condensate. For \hat{T}_+ we find a similar expression in terms of spherical Bessel functions.

In the collective model discussed here, these spatial matrix elements are folded with a distribution function $g(\beta) = \beta^2 \exp(-\beta/a)/2a^3$ for the charge and the magnetization along the string. With this distribution we reproduce the observed dipole form for the electric form factor of the proton $G_E^p = 1/(1 + k^2a^2)^2$. The scale parameter a can be determined from the proton charge radius. The helicity amplitudes for a given baryon resonance can be obtained by combining the spatial contribution with the appropriate spin-flavor matrix elements [3]. For example, the proton helicity amplitudes for the $N(1520)D_{13}$ resonance are given by

$$\begin{aligned} A_{1/2} &= 2i\mu \sqrt{\pi/k_0} [m_3 k_0 a / g_3 - k^2 a] / (1 + k^2 a^2)^2, \\ A_{3/2} &= 2i\mu \sqrt{3\pi/k_0} m_3 k_0 a / g_3 (1 + k^2 a^2)^2. \end{aligned} \quad (6)$$

Just as in the harmonic oscillator quark model, the asymmetry parameter $A = (A_{1/2}^2 - A_{3/2}^2)/(A_{1/2}^2 + A_{3/2}^2)$ changes rapidly from -1 to $+1$ with increasing momentum transfer. For small values of k the helicity-1/2 amplitude is small because of the canceling contributions of the electric and magnetic terms, whereas for large values of k the contribution of the magnetic term dominates. However, in the collective model the helicity amplitudes fall as powers of the momentum transfer, in contrast to the exponential decrease of harmonic oscillator form factors. It is important to note that this property holds for all baryon resonances. Further improvements can be obtained by the introduction of spin-flavor symmetry breaking and the stretching of the strings [4].

4 Summary and conclusions

In conclusion, we have discussed electromagnetic form factors of nonstrange baryon resonances in the context of a collective model of the nucleon. Collective form factors are obtained by folding with a probability distribution, which is determined by the elastic form factor of the proton. Within the assumptions of the model the transition form factors to all other nonstrange baryon resonances are derived in closed form and are predicted to drop as powers of the momentum transfer. Such predictions of electromagnetic properties can be tested in future photo- and electroproduction experiments.

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Figure 1: Collective model of baryons and its idealized string configuration (the charge distribution of the proton is shown as an example).

